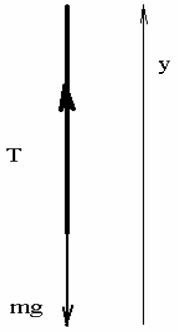
Physics 151 Class Exercise: Centripetal Acceleration - KEY

For each of the following situations, draw a free body diagram showing all of the forces acting on the object. Then write an equation expressing the sum of forces in the radial direction – from the center of the circle to the object and **write an algebraic expression for the centripetal acceleration a_c in terms of the given variables.**

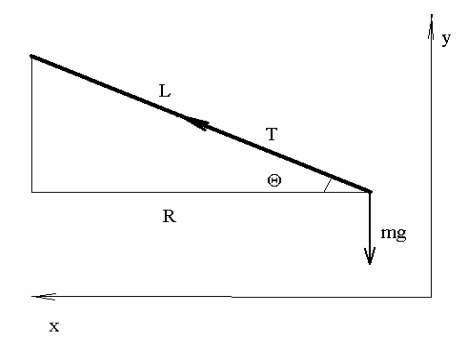
Situation #1 – A boy ties a ball to the end of a string (length L) and then swings the ball in a vertical circle. Consider the ball when it is at the lowest point in its swing, the tension in the string is T at that moment.



In this situations all forces act along one direction y and centripetal acceleration points up, so that $ma = T - mg$ or

$$a_c = \frac{T}{m} - g$$

Situation #2 – The boy swings the ball (mass m) in a horizontal circle (radius R) about his head. (Hint: How does gravity affect the direction of the tension in the string?)



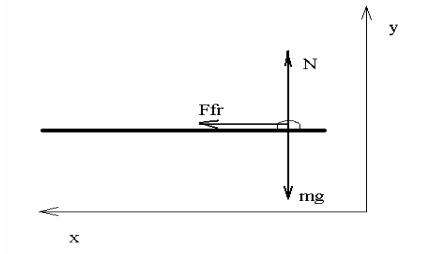
The direction of tension makes angle θ with the horizontal. The centripetal acceleration points along axes x to the center of the circle, because of that one has

x: $T\cos\theta = ma$ $\quad ma = T\cos\theta$

y: $T\sin\theta - mg = 0$, $\quad mg = T\sin\theta$, dividing second equation by the first gives $g/a = \tan\theta$, $a = g/\tan\theta$, the angle θ itself can be found as $\theta = \arccos(R/L)$, so finally $a_c = g/\tan\left(\arccos\left(R/L\right)\right)$.

Situation #3 – A bug sits on the edge of a 45 record being played on a turntable, the coefficient of the static friction between the bug and a record is μ.

The force, which provides the centripetal acceleration along axes x is the friction force F_{fr},

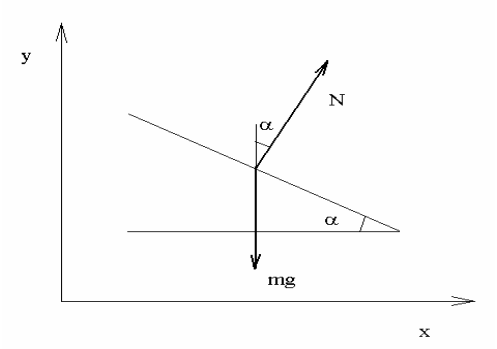


then $\begin{matrix} x: & ma = F_{fr} \\ y: & N - mg = 0 \end{matrix}$, since $F_{fr} = \mu N$, one has

$ma = \mu N$

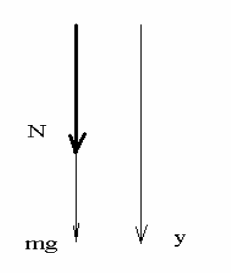
$mg = N$ or $ma = \mu mg$, $a_c = \mu g$

Situation #4 -- A car moves on a circular exit ramp banked at angle α to the horizontal. Neglect friction. (Hint: Draw the car so that you are looking at the back or the front.)
In this situation centripetal acceleration will be along the x-axis as shown in the picture, so that



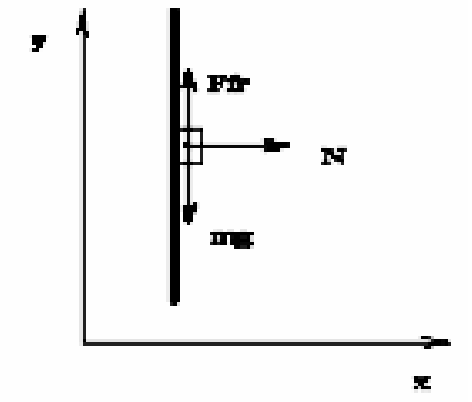
x: $ma = N\sin\alpha$
y: $N\cos\alpha - mg = 0$ or $ma = N\sin\alpha$, $mg = N\cos\alpha$, dividing the first equation by the second one can find
$a/g = \tan\alpha, \quad a_c = g\tan\alpha$

Situation #5 – A "corkscrew" roller coaster does a loopty-loop. Consider the situation of a man of mass m riding in the roller coaster when he is upside-down, the normal force acting on the man is N.
In this situation all the forces and centripetal acceleration point down along axes y, so that



$ma = N + mg$ or $a_c = \frac{N}{m} + g$

Situation #6 - A daredevil stunt involves riding a motorcycle around the vertical inside wall of a cylindrical structure. The coefficient of friction between motorcycle and a wall is μ (Hint: How big does the normal force have to be?)
In this situation the normal force N provides the centripetal acceleration in positive x direction, so



x: $ma = N$
y: $F_{fr} - mg = 0$, since $F_{fr} = \mu N$, one has
$ma = N$
$\mu N = mg$ or $\mu ma = mg, \quad a_c = \frac{g}{\mu}$